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How Many Factors? Does Adding Momentum and Volatility Improve Performance?

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Abstract: This paper considers whether adding two established anomalies, momentum and low volatility, will improve our understanding of asset pricing beyond the FF5 model. We do this by considering whether these factors provide economic, as opposed to statistical, significance within the asset pricing model. We measure economic significance in two ways: First, we consider whether the factor coefficient signs and values on the factors are economically meaningful, for example, do the coefficients distinguish between high- and low-risk portfolios? Second, we consider an out-of-sample trading rule based on expected returns derived from each asset pricing model. Our results suggest that the momentum and volatility factors provide no additional information over the FF5 model. Moreover, it is not clear that the FF5 model itself provides a noticeable improvement over the FF3 model. Of note, the momentum and low-volatility factors exhibit limited statistical significance and have similar coefficients across high and low values of different anomalies and big- and small-firm portfolios. The trading performance of a seven-factor model, while reasonable itself, is worse than both the FF3 and FF5 models. Furthermore, based on the trading results, the FF5 model provides no noticeable contribution over the FF3 model, the latter of which could be regarded as preferred.

Keywords: five-factor model; momentum; volatility; time variation; expected returns

JEL Classification: C22; G12

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1. Introduction

The five-factor model (FF5, Fama and French, 2015) is presented as the leading model at the head of the current literature on asset pricing. This model builds on the previous three-factor version that includes the market, size, and value factors by further including a profit and investment factor. Inevitably, such a model will ignore other factors, such as the widely established momentum and low-volatility factors (e.g., Cahart, 1997; Ang et al., 2006, respectively), whose inclusion could then be considered as part of a seven-factor model (F7). Moreover, as highlighted by Harvey et al. (2016), there exists a wide range of factors that could potentially be incorporated into any asset pricing model and increasing the number of factors. However, this in turn leads to the danger of a factor zoo (Cochrane, 2011) in which any model designed to explain asset pricing becomes subject to many regressors, with factors included to account for different stock return characteristics. This, in turn, brings its own econometric problems (e.g., multicollinearity and model stability). Thus, care needs to be exercised when deciding whether further factors should be added to our asset pricing model. Moreover, such an issue is an empirical one as the underlying theoretical rationale for the factors remains unclear.

In this paper, we consider whether incorporating momentum and volatility into the five-factor model can improve the performance of the asset pricing model. While one approach is to consider the statistical significance of individual variables, our primary method of investigation is to consider the economic significance of these factors by examining the nature of the factor coefficients and to derive a trading-based rule for alternative asset pricing models (e.g., CAPM, FF3, FF5 and F7) and compare them to a more passive investment, i.e., holding a portfolio style. Should the factors contain useful information the asset pricing model-based trading portfolio should then outperform a passive approach. Moreover, in generating our trading rule, we seek to take advantage of time variation within the asset pricing model and the changing strength of the asset pricing factors. Thus, in this paper, a second key area of interest is the nature of time variation within asset pricing factors and how this affects their economic meaning in relation to the predictive ability for stock returns.

Our analysis begins by considering the five-factor model for a range of different style portfolio types. Our interest centres on whether the factors appear to load correctly across the different portfolios, and whether the constant term that represents potential abnormal returns is statistically zero. We then seek to consider whether adding factors designed to capture the momentum and the low-volatility anomaly are significant within the asset pricing model. In addition, we recognise that the nature of the factor model will be time-varying. Thus, we seek to examine this time variation and consider whether it can be used in developing a trading model. To do this, we undertake a series of five-year rolling regressions and examine the behaviour of the coefficients. Notably, we consider whether the coefficient signs change over the sample period. Such a change would impact the economic meaning of the factor, for example, the size (small minus big firms) factor should always have a positive coefficient of small-firm portfolios. Should this not occur, that casts doubt on the usefulness of that factor. Knowledge of the time-varying factors can ultimately be used to generate estimates of expected returns, which we can then be utilised in a trading rule to further establish the economic content of the asset pricing model and the factors used.

It is hoped that the results in this paper will be of interest to a wide range of people. For academics, this will help in moving closer to understanding the behaviour of asset prices. For investors, this will present information as to whether the factors identified in academic work have implications regarding investment decisions. For corporate managers, asset pricing models form the basis of cost-of-capital calculations, thus, the results here will indicate whether such models require adjustment for additional factors.

2. Five-Factor Model

We begin by examining the ability of the Fama and French (2015) five-factor model to explain the behaviour of a range of stock return portfolios. We obtain portfolios that are first separated between large and small firms. These two broad portfolio types are then separated across book-to-market, profitability, investment, momentum, short- and long-term reversal and quality. The data is obtained monthly from the data library of Ken French (except the quality factor) and is sampled over the period 1990:1–2016:10.[1]

We first estimate the standard factor model as follows:

$$r_t - r_f = \alpha + \beta_m\ (r_{mt} - r_f) + \beta_s\ SMB_t + \beta_h\ HML_t + \beta_p\ PMU_t + \beta_i\ CMA_t + \varepsilon_t, \tag{1}$$

where $r_t - r_f$ is the risk premium on the portfolio of interest, $(r_{mt} - r_f)$ refers to the market risk premium, while SMB_t, HML_t, PMU_t, and CMA_t are hedged portfolios for size (small minus big firms), value (high minus low book-to-market firms), profitability (profitable minus unprofitable firms) and investment (conservative minus aggressive firms), respectively.

We estimate this model for the double-sorted portfolios noted above, where Table 1 contains the results for the large firms and Table 2 for the small firms. Hence, Table 1 reports the results of Equation (1) for large firms across high, medium, and low book-to-market, profitability, investment, momentum, reversal, and quality characteristics, with Table 2 reporting likewise for small firms. Thus, we are considering a range of different portfolio types and asking whether the FF5 model can provide a reasonable characterisation of them. Fama and French (1996) note that the three-factor model can be reinterpreted as either a multifactor intertemporal CAPM or APT model and can explain the behaviour of anomalies beyond size and value. Hence, these additional factors should be able to explain a wider range of stock return portfolios rather than those they directly relate to (i.e., the value factor should explain portfolios other than just book-to-market ones). Of interest, Fama and French note that the three-factor model is unable to account for the momentum effect. Hence, we continue this approach by considering the ability of the five-factor model to account for the behaviour of a range of portfolios beyond the portfolio that directly relates to the FF5 factors and, thus, including momentum.

Table 1 reports the estimation results of the FF5 model for 21 portfolios across seven investment styles for large firms. An examination of this table reveals that the alpha (intercept) is statistically significant for 13 of the 21 portfolios considered. This suggests that there remains a part of the portfolio return that the FF5 model does not explain. In terms of the market factor, the estimated parameter is statistically significant across all portfolios and varies around 1. In considering the parameters for which the market beta is above 1, we can see this occurs for high book-to-market, low profitability, low momentum, low short-term reversal, and low-quality stocks. Thus, the portfolios identified as exhibiting greater than market risk would appear to coincide with those we would reasonably think of as risky (i.e., a low stock price relative to book value, low profits, and so on). For the investment and long-term reversal portfolios, however, the value of beta is similar for both the high and low portfolios. This indicates little economic information arising from these coefficient values for these portfolios.

Examining the size factor, we would expect this to load significantly negative on all portfolios as they are formed of large stocks. However, this factor is only significant for 15 of the 21 portfolios. Furthermore, it is positive (although insignificant) for four portfolios. Thus, although most of the large-firm portfolios reveal an appropriate size effect, against expectations, this is not universal. The value factor suggests that the low portfolios for profit, momentum, long-term reversals, and quality have a value loading with, typically, a growth bent for the equivalent high portfolios. The investment and short-term reversal portfolios appear to have no obvious value or growth loading as the value factor exhibits similar coefficient values for the high and low portfolios. Hence, this factor is unable to discriminate between the risk characteristics of these portfolios. Comfortingly, the value portfolio does indeed exhibit a positive value factor loading.

[1] mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. The quality factor is obtained from https://www.aqr.com/library/data-sets/quality-minus-junk-factors-monthly.

Portfolios	Factors					
	Constant	**Market**	**SMB**	**HML**	**Profit**	**Investment**
High BM	0.132 (1.52)	1.095 (29.34)	−0.093 (−1.48)	0.774 (9.35)	−0.035 (−0.38)	−0.055 (−0.51)
Mid BM	0.110 (1.47)	1.017 (41.31)	−0.112 (−2.96)	0.329 (6.76)	0.198 (3.60)	0.160 (3.14)
Low BM	0.283 (5.84)	0.990 (72.97)	−0.135 (−6.10)	−0.341 (−14.21)	0.160 (6.84)	0.052 (1.48)
High Profit	0.266 (5.91)	0.984 (74.06)	−0.130 (−4.95)	−0.169 (−7.09)	0.332 (12.72)	0.059 (1.88)
Mid Profit	0.154 (3.52)	1.001 (60.29)	−0.119 (−4.25)	0.077 (1.81)	−0.013 (−0.30)	0.087 (1.57)
Low Profit	0.266 (3.87)	1.048 (45.84)	−0.134 (−5.11)	0.147 (4.06)	−0.576 (−8.05)	−0.182 (−3.76)
High Inv	0.371 (6.01)	1.030 (64.06)	−0.073 (−3.17)	−0.066 (−2.13)	−0.019 (−0.71)	−0.510 (−11.36)
Mid Inv	0.222 (5.02)	0.967 (60.10)	−0.156 (−7.19)	0.010 (0.41)	0.141 (5.62)	0.225 (8.10)
Low Inv	0.116 (1.91)	1.026 (46.41)	−0.122 (−5.49)	−0.052 (−1.27)	0.156 (2.34)	0.656 (9.91)
High Mom	0.432 (3.83)	0.963 (20.44)	−0.086 (−1.13)	−0.112 (−1.17)	0.068 (0.67)	0.047 (0.29)
Mid Mom	0.094 (1.21)	0.975 (46.76)	−0.088 (−2.70)	0.095 (2.45)	0.281 (4.49)	0.111 (1.70)
Low Mom	−0.192 (−1.02)	1.239 (16.54)	0.193 (1.43)	0.353 (2.21)	0.193 (0.99)	−0.263 (−1.05)
High STR	0.197 (1.91)	0.946 (24.41)	−0.191 (−3.11)	−0.053 (−0.89)	−0.106 (−0.99)	−0.021 (−0.20)
Mid STR	0.256 (4.75)	0.991 (49.63)	−0.147 (−4.77)	−0.035 (−1.08)	0.145 (3.55)	0.119 (2.53)
Low STR	0.110 (0.84)	1.185 (23.70)	0.108 (1.43)	0.019 (0.19)	0.044 (0.41)	−0.039 (−0.27)
High LTR	0.244 (3.48)	1.030 (56.56)	−0.164 (−6.33)	−0.229 (−5.60)	0.180 (4.99)	−0.056 (−0.93)
Mid LTR	0.209 (4.34)	0.946 (58.02)	−0.087 (−2.75)	0.177 (4.29)	0.204 (6.77)	0.217 (4.01)
Low LTR	0.358 (2.78)	1.036 (26.96)	0.082 (1.35)	0.274 (3.85)	−0.039 (−0.56)	0.319 (2.79)
High Qual	0.099 (2.34)	0.923 (65.96)	−0.119 (−5.21)	−0.283 (−12.30)	0.156 (5.59)	0.029 (0.95)
Mid Qual	−0.074 (−1.60)	1.038 (59.17)	−0.083 (−3.35)	0.153 (5.18)	0.074 (2.56)	0.067 (1.74)
Low Qual	−0.165 (−2.99)	1.131 (53.71)	0.016 (0.51)	0.370 (9.60)	−0.432 (−13.58)	−0.198 (−4.29)

Notes: Entries are coefficient values (and Newey–West t-statistics) for Equation (1) for large-firm portfolios. The portfolios are book-to-market, profitability, investment, momentum, short-term reversal, long-term reversal, and quality. The factors are the market portfolio, small minus big, value minus growth, high profit minus low profit, and low investment minus high investment.

Table 1 Large-Firm Portfolios.

Portfolios	Factors					
	Constant	Market	SMB	HML	Profit	Investment
High BM	0.293 (3.80)	1.002 (40.78)	0.908 (21.77)	0.578 (11.91)	0.068 (1.00)	0.177 (2.49)
Mid BM	0.300 (3.87)	0.959 (33.65)	0.835 (21.78)	0.217 (4.18)	0.090 (1.04)	0.189 (1.80)
Low BM	0.118 (1.36)	1.016 (27.20)	1.026 (24.87)	−0.307 (−6.31)	−0.264 (−2.31)	−0.006 (−0.05)
High Profit	0.189 (2.99)	1.070 (54.34)	0.959 (28.17)	0.271 (5.99)	0.543 (13.27)	−0.074 (−0.90)
Mid Profit	0.251 (3.26)	0.959 (37.75)	0.844 (21.79)	0.226 (4.35)	0.272 (4.21)	0.133 (1.48)
Low Profit	0.190 (2.12)	1.007 (29.99)	0.963 (21.27)	−0.045 (−0.84)	−0.549 (−4.97)	0.168 (1.70)
High Inv	0.112 (1.69)	1.034 (39.79)	0.957 (23.77)	0.075 (1.48)	−0.110 (−1.48)	−0.299 (−2.96)
Mid Inv	0.330 (4.38)	0.915 (29.38)	0.821 (20.50)	0.202 (4.31)	0.118 (1.32)	0.244 (3.23)
Low Inv	0.347 (3.25)	1.038 (29.63)	1.007 (23.15)	0.063 (1.16)	−0.285 (−2.30)	0.535 (5.51)
High Mom	0.625 (4.01)	0.986 (18.02)	0.922 (9.93)	−0.084 (−0.75)	−0.063 (−0.45)	0.126 (0.70)
Mid Mom	0.203 (3.26)	0.962 (57.83)	0.835 (30.64)	0.290 (5.44)	0.274 (9.72)	0.105 (1.98)
Low Mom	−0.274 (−1.62)	1.247 (16.03)	1.226 (9.31)	0.405 (2.75)	−0.031 (−0.17)	−0.205 (−0.86)
High STR	0.127 (0.97)	0.984 (26.40)	0.897 (15.69)	0.018 (0.27)	−0.191 (−1.81)	0.153 (1.38)
Mid STR	0.207 (3.19)	0.992 (58.17)	0.884 (28.37)	0.267 (5.27)	0.244 (6.62)	0.054 (1.01)
Low STR	0.196 (1.55)	1.225 (28.12)	1.179 (17.81)	0.159 (2.67)	0.037 (0.28)	−0.226 (−2.33)
High LTR	0.059 (0.69)	1.086 (41.75)	0.918 (23.39)	0.106 (2.06)	0.254 (3.76)	−0.037 (−0.38)
Mid LTR	0.257 (3.82)	0.951 (51.56)	0.788 (26.61)	0.274 (5.79)	0.242 (8.98)	0.173 (3.08)
Low LTR	0.318 (3.36)	1.118 (39.42)	1.146 (32.25)	0.325 (7.05)	−0.062 (−1.62)	0.272 (5.39)
High Qual	0.149 (3.13)	0.939 (80.83)	0.906 (42.88)	0.091 (2.67)	0.305 (10.86)	0.028 (0.62)
Mid Qual	−0.047 (−1.34)	1.014 (77.92)	0.922 (50.04)	0.294 (9.87)	0.141 (6.35)	0.018 (0.40)
Low Qual	−0.362 (−4.69)	1.196 (54.91)	1.106 (34.38)	0.224 (4.14)	−0.479 (−8.10)	−0.199 (−1.93)

Notes: Entries are coefficient values (and Newey–West t-statistics) for Equation (1) for large-firm portfolios. The portfolios are book-to-market, profitability, investment, momentum, short-term reversal, long-term reversal, and quality. The factors are the market portfolio, small minus big, value minus growth, high profit minus low profit, and low investment minus high investment.

Table 2 Small-Firm Portfolios.

For the profit and investment factors, these are statistically significant for 13 and 9 portfolios respectively (11 and 6 times if we exclude the portfolios directly linked to the factors). Thus, these factors are significant for less than half of the portfolios (after excluding the portfolios related to the factors). In terms of assigning an economic explanation for these factors, we can see that they often load negatively on those portfolios that load positively on the value factor. Hence, this presents a consistent view in which potentially distressed stocks (high book-to-market) are also characterised by low profitability. However, this pattern does not appear for the momentum and reversal portfolios.

Table 2 presents the same set of regression results but, this time, for small firms and for which we can make similar observations. Of note, for 14 of the 21 portfolios, the alpha term is statistically significant, again, suggesting the presence of unexplained return to these portfolios. In terms of the market portfolio, the generally accepted view is that small firms are riskier than large firms and, so, we would except this parameter to be larger. The evidence in Table 2 could be argued to generally support this view as the coefficient of the market portfolio is higher for 14 of the 21 portfolios compared to the equivalent large-firm portfolio. However, the difference in coefficient values between Tables 1 and 2 is small, suggesting little discrimination in the risk characteristics between the portfolios. Moreover, for some key portfolios, such as high book-to-market and low profitability, the market coefficient is lower for the small-firm portfolio compared to the large-firm portfolio. The size factor is positive and significant for all portfolios, which is consistent with all these portfolios containing small firms. In terms of the value factor, consistent with Table 1, low momentum, reversals, and quality have a value loading. However, the high-profit portfolio also loads positively on value, in contrast to Table 1, where low profit loads positively. The profit factor is significant for 12 portfolios, while the investment factor is significant for 8, although excluding the portfolios that directly relate to the factors, this falls to 9 and 5 respectively. Hence, these newer factors—that are the difference between the three- and five-factor models—are, at best, significant for only half the considered portfolios.

Overall, these results suggest a few pertinent points. First, the FF5 model does not fully explain the movement in a range of stock return portfolios.[2] Second, the additional factors in the FF5, compared to the FF3 model, do not capture movement in a wide range of portfolios beyond those directly related to them. Third, the nature of the coefficients attached to the factors lacks consistency with respect to the economic interpretation ascribed to them. For example, we would expect riskier portfolios to exhibit a higher market beta. Equally, we would expect consistency between factors (e.g., size) and its effect on equivalent portfolios (e.g., those composed of large or small stocks). While this is mostly the case, however, it is not universally so. The nature of these results, therefore, appears to leave open the issue of whether the asset pricing model can be improved through the addition of two factors (momentum and low volatility) that have been widely discussed within the literature.

3. Momentum and Low-Volatility Factors

From the above FF5 regression results, what we observe is that the factors are not significant across all the different portfolios. We also note that the alpha (intercept) term is statistically significant for several portfolios. This leads to the question of whether we need to include additional factors. However, we also wish to avoid the factor zoo issue, where we increase the number of factors continuously to match the in-sample characteristics of the data. Hence, to avoid this issue, we consider only two further factors, both of which are widely discussed within the literature: momentum and volatility. The momentum factor is defined as a portfolio of winner stocks against loser stocks, while the volatility factor is a portfolio of high volatility stocks minus a portfolio of low-volatility stocks. Specifically, momentum is defined by the stock's performance over the past year, with winners defined as above median performance and losers below median performance. For the volatility factor, we consider several alternatives, including the standard deviation, beta, and idiosyncratic volatility. In the reported results, we use the standard deviation with the results for the other measures that are qualitatively similar. We define 10 portfolios based on the

[2] Of course, it is arguable that no model will ever capture all the dependencies within a wide range of series.

ranked standard deviation and use the difference between the highest and lowest standard deviation portfolios as the volatility factor.

In assessing the additional factors, our key approach is to examine whether such factors have economically meaningful parameters and whether they enhance the out-of-sample performance of the asset pricing model and, thus, have information useful to investors rather than just matching statistical behaviour. Table 3 presents the estimation results for the F7 model (i.e., the FF5 plus momentum and volatility). For the sake of brevity, we only report results for the two additional factors across 14 portfolios, and we consider only the high and low portfolios.[3]

Large Firms					
Portfolios	**Volatility**	**Momentum**	**Portfolios**	**Volatility**	**Momentum**
High BM	0.008 (0.51)	−0.095 (−2.89)	Low Mom	−0.020 (−1.31)	−0.661 (−23.95)
Low BM	−0.009 (−1.55)	0.001 (0.04)	High STR	0.001 (0.03)	−0.002 (−0.04)
High Profit	−0.010 (−1.18)	0.015 (1.23)	Low STR	0.013 (0.47)	−0.033 (−0.52)
Low Profit	0.010 (1.33)	−0.042 (−1.99)	High LTR	0.006 (0.67)	0.003 (0.14)
High Inv	0.004 (0.47)	0.003 (0.25)	Low LTR	−0.016 (−0.92)	0.056 (1.51)
Low Inv	−0.027 (−3.31)	−0.045 (−1.92)	High Qual	−0.001 (−0.17)	0.020 (1.33)
High Mom	0.012 (1.41)	0.372 (15.31)	Low Qual	0.010 (1.40)	−0.001 (−0.61)
Small Firms					
High BM	−0.009 (−0.91)	0.101 (5.15)	Low Mom	0.012 (1.03)	−0.564 (−20.55)
Low BM	−0.034 (−2.74)	0.123 (3.87)	High STR	−0.021 (−1.05)	0.063 (1.08)
High Profit	−0.007 (−0.70)	0.073 (2.88)	Low STR	−0.008 (−0.47)	−0.095 (−1.67)
Low Profit	−0.027 (−2.15)	0.130 (4.64)	High LTR	−0.012 (−0.92)	−0.118 (−5.32)
High Inv	−0.030 (−2.60)	0.103 (3.25)	Low LTR	−0.002 (0.19)	0.021 (1.27)
Low Inv	0.001 (0.06)	0.151 (5.54)	High Qual	−0.010 (−1.79)	0.014 (0.88)
High Mom	−0.029 (−2.78)	0.429 (25.37)	Low Qual	0.016 (1.28)	−0.058 (−2.73)

Notes: Entries are coefficient values (and Newey–West t-statistics) for Equation (1) for large- and small-firm portfolios. The portfolios are book-to-market, profitability, investment, momentum, short-term reversal, long-term reversal, and quality. The factors are the winners minus losers and high minus low volatility. These are in addition to the factor notes in Tables 1 and 2.

Table 3 Adding the Volatility and Momentum Factors.

The results from this table present an interesting view. Regarding the volatility anomaly, this factor is only statistically significant once for large stocks (negatively for the low investment portfolio) and four times for small

[3] Remaining results are available upon request.

stocks. This suggests that the volatility factor has very little explanatory power for stocks. For small stocks, volatility loads negatively on low book-to-market and profit stocks and high investment and momentum stocks. Recalling that the volatility factor is defined as a high-volatility portfolio minus a low-volatility portfolio, this implies relatively lower volatility within these portfolios. In terms of the momentum factor, for large stocks, it equally has very little explanatory power, being significant for only four portfolios. However, two of those are the momentum portfolios and, thus, the momentum factor only provides additional significance for the high book-to-market portfolio and the low profit portfolio (both have negative coefficients). For small stocks, 10 portfolios have a significant momentum factor, including the two momentum portfolios. In principle, this suggests a greater amount of information flowing from the momentum factor. However, even for those portfolios with a significant momentum factor, the loadings are broadly similar across high and low portfolio variants, suggesting little economic meaning behind the statistical significance. For example, the coefficient values are 0.101 and 0.123 for the high and low book-to-market portfolios respectively.

These results suggest that the volatility and momentum factors do not appear to have explanatory power for stock returns beyond their own portfolios. This raises the question of whether they should be included in an asset pricing model. Specifically, any asset pricing factor should have the ability to explain a range of stock return characteristics, as suggested in Fama and French (1996). This is necessary to avoid the factor zoo, where factors are included to fit each individual stock return characteristic. That said, the nature of the above results is largely statistical, while our key interest in whether factors should be included in the asset pricing model is whether they contain economic power. We consider this in two ways. First, we examine the time-varying nature of the coefficient and whether its sign is consistent with the intended economic interpretation of the factors.[4] Second, we consider a trading rule, which allows us to examine whether the inclusion of additional factors would aid investors decision-making.

4. Time Variation

Inevitably, over any reasonable time period, the coefficients within a factor regression model are unlikely to remain constant. Furthermore, it is important to understand how these factor loadings vary over time and whether this variation is related to economic conditions. For factors to have any economic meaning, their movement should be related to economic fundamentals that proxy for risk; otherwise, it is unclear what information the factor conveys. Moreover, it may be possible to use information contained within this time variation to improve portfolio performance by incorporating this variation into a trading rule based on expected returns.

To examine time variation within the factor coefficients, we re-estimate Equation (1) for each of our portfolios using a five-year moving fixed window. To illustrate the time variation within factor coefficients, Figure 1 presents the plots for the large firm—a high book-to-market portfolio regression that includes the seven factors (i.e., F7: FF5 plus volatility and momentum).[5] Evident within this figure is a noticeable amount of time variation for all the factors across the sample period. Moreover, and more importantly, these results show that the key nature of the coefficients change over the sample period. For the market factor, we can see that its value changes from >1 over the beginning and middle portion of the sample to <1 towards the end of the sample. Thus, the nature of this portfolio changes from aggressive to defensive. Knowledge of such a change is obviously important to an investor who will set a level of risk to achieve a desired return. A change in the nature of beta (systematic risk) will inevitably affect that (expected) return.[6]

[4] Of course, we can also examine the reported static coefficients which, as noted, are not supportive of these additional factors being able to separate high and low risk portfolios as their values are similar across portfolios.

[5] The remaining plots are available upon request.

[6] Examining the plot would appear to suggest that large value stocks became less risky during the recent financial crisis period. Of interest, no equivalent fall is noted for the previous (dotcom) crisis period.

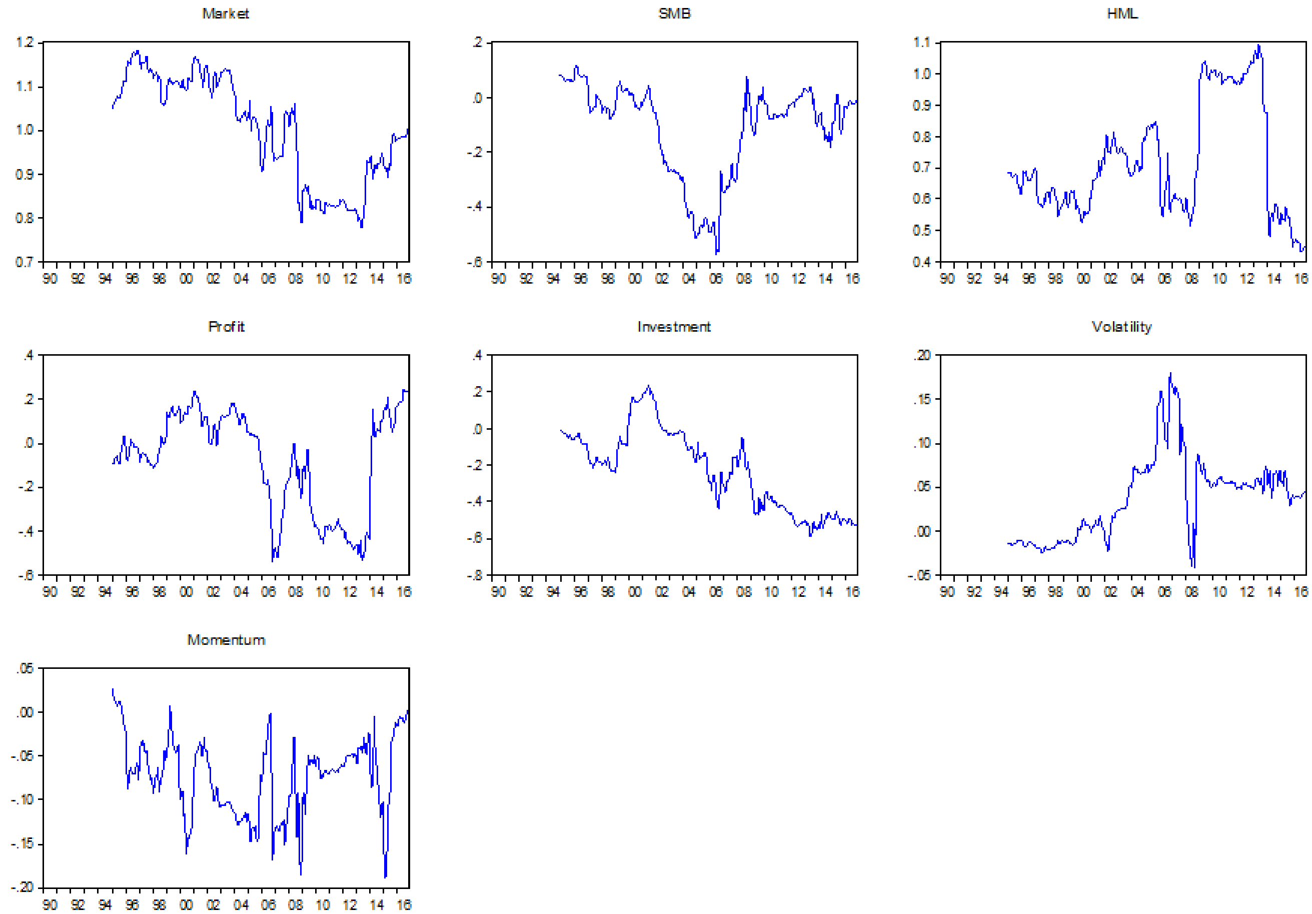

Figure 1 Time-Varying Factor Coefficients.

As this portfolio contains large stocks, we would expect the SMB factor to load negatively on the portfolio. However, while this is mainly the case, we can see periods at the beginning and end of the sample where the SMB coefficient is positive, albeit for short-lived periods of time. Nonetheless, again, this has implications for an investor seeking a large-firm portfolio given its perceived different risk profile compared to a small-firm portfolio. Equally, we can see a change in the coefficient sign for the profit, investment, volatility, and momentum factors, albeit that for this latter factor, the coefficient is positive only over a very brief period of time. In addition to the change in sign, we can observe large changes in the magnitude of the coefficient for all factors. For example, as a value portfolio, we would expect to see the value factor exhibit a positive coefficient throughout. While this is the case, we can observe the HML factor change from around 0.55 in mid-2008 to 1 by the end of 2008, remain high, and then exhibit an equally dramatic fall occurring towards the end of 2013 (i.e., an elevated coefficient over the recent crisis period). Similar large coefficient changes can be observed for the other factors. These changes in both coefficient sign and value do suggest instability in the factors and casts doubt on their economic content. Notably, the momentum factor appears highly volatile and cycles between close to 0 and around −0.15 several times over the sample.

To provide an overview of the time variation across all the factors, Table 4 reports the mean coefficient value as well as the minimum and maximum values. Evident from this table is that the characteristics observed in the graph discussed above are also true for most of the other portfolios. We can observe, for the market factor, the value of this coefficient transcending 1 for all portfolios considered here.[7] Equally, we can see the coefficient values crossing 0 for most of the factors across the portfolios, with the exception to this being the factor directly related to

[7] We only consider a subset of the portfolios in this table. However, the remaining portfolios exhibit the same characteristics and are available upon request.

the portfolio under question, e.g., the HML factor for high and low book-to-market portfolios and so forth. However, even here there are some exceptions particularly for small firms where the low book-to-market, high profit, and low investment portfolios all exhibit signs that are opposite to that expected based on the construction of the factor.

	Factors						
Portfolios	**Large Firms**						
	Market	**SMB**	**HML**	**Profit**	**Investment**	**Volatility**	**Momentum**
High BM	1.003 0.78:1.03	−0.115 −0.57:0.12	0.717 0.43:1.09	0.229 −0.59:0.24	0.079 −0.54:0.25	0.037 −0.04:0.18	−0.073 −0.19:0.03
Low BM	0.974 0.91:1.08	−0.137 −0.27:−0.04	−0.324 −0.46:−0.23	0.135 −0.09:0.39	0.060 −0.12:0.25	−0.007 −0.07:0.03	0.004 −0.08:0.08
High Profit	0.987 0.91:1.09	−0.131 −0.30:−0.06	−0.143 −0.26:−0.08	0.315 0.18:0.49	0.053 −0.12:0.17	−0.006 −0.03:0.03	−0.002 −0.09:0.06
Low Profit	0.994 0.89:1.13	−0.131 −0.28:0.01	0.072 −0.08:0.25	−0.688 −0.99:−0.47	−0.132 −0.035:0.04	0.020 −0.03:0.06	−0.022 −0.15:0.13
High Inv	1.005 0.91:1.12	−0.084 −0.26:0.04	−0.046 −0.20:0.17	0.011 −0.27:0.17	−0.512 −0.84:−0.28	−0.001 −0.05:0.06	0.006 −0.07:0.10
Low Inv	1.030 0.92:1.16	−0.089 −0.23:−0.02	−0.042 −0.23:0.19	0.061 −0.06:0.27	0.636 0.43:0.81	−0.024 −0.06:0.02	−0.035 −0.14:0.05
	Small Firms						
High BM	1.022 0.89:1.16	0.961 0.78:1.21	0.677 0.29:1.03	0.116 −0.19:0.37	0.040 −0.020:0.23	−0.006 −0.06:0.05	0.072 −0.07:0.18
Low BM	1.063 0.94:1.20	1.054 0.89:1.22	−0.193 −0.64:0.17	−0.111 −0.042:0.31	−0.302 −0.59:0.08	−0.019 −0.08:0.07	0.079 −0.07:0.19
High Profit	1.074 0.95:1.20	1.002 0.83:1.18	0.173 −0.20:0.60	−0.186 −0.38:0.23	−0.397 −0.72:−0.03	−0.016 −0.07:0.09	0.065 −0.06:0.19
Low Profit	1.049 0.95:1.19	1.008 0.86:1.14	0.169 −0.14:0.38	−0.237 −0.59:−0.03	0.457 0.11:0.68	0.008 −0.07:0.07	0.106 −0.02:0.23
High Inv	1.063 0.97:1.21	0.998 0.83:1.21	0.304 −0.06:0.69	0.419 0.18:0.82	−0.122 −0.42:0.29	0.009 −0.07:0.09	0.064 −0.11:0.14
Low Inv	1.056 0.93:1.19	0.998 0.86:1.12	0.089 −0.27:0.40	−0.578 −0.82:−0.34	0.063 −0.19:0.37	−0.017 −0.07:0.06	0.085 −0.11:0.20

Notes: Entries are the mean, minimum, and maximum coefficient values for Equation (1) estimated using a five-year rolling window. The portfolios are book-to-market, profitability, and investment. The factors are the market portfolio, small minus big, value minus growth, high profit minus low profit, and low investment minus high investment.

Table 4 Summary Statistics of 5-Year Moving Average for Coefficients.

The pertinent point arising from this analysis is in trying to understand the nature of the economic relation that lies behind the factor model. Factors are included as they proxy for an aspect of risk that affects a given portfolio. For the coefficient of that risk factor to change, this suggests that the nature of the risk relation changes such that, for example, a factor that positively contributes to risk of one part of the sample now mitigates risk in another part. This makes an economic explanation for the factors difficult unless the factor can be linked to an explicit economic variable.

As such, having examined the presence of time variation within the expected return asset pricing model, we consider whether this time variation is linked to economic or market conditions. To do this, we estimate a model that regresses the time-varying factors against both the growth rate of industrial production and the level of the VIX (the implied volatility index). We choose these two variables as a way of capturing both macroeconomic and market risk. As stock market factors are believed to proxy for movements in expected returns, we would expect such risk measures to impact on the behaviour of these factors. An increase in economic risk, as indicated by negative output growth, or an increase in market risk, as indicted by an increase in VIX, should be associated with an increase in expected returns. Hence, where a fall in industrial production indicates an increase in risk, we would expect to see a negative correlation with the coefficients on the market, SMB, HML, investment, and volatility factors, as higher values are associated with greater risk. For the profit factor, we would expect to see a positive relation, while the relation for the momentum factor is undefined. For the VIX, we would expect to see the opposite signs to those noted for output growth as an increase in VIX is associated with greater risk.

Table 5 presents the estimation results of the time-varying factor parameters on the first lag of the change in industrial production and VIX for the large-firm portfolios. Taking an overall view, we can see that the coefficient of industrial production growth varies between positive and negative values, and significance and insignificance, across the different portfolios. Similar behaviour is found for the VIX coefficient across the different portfolios. This suggests the lack of a consistent pattern across all portfolios. Nonetheless, we can observe, in general, that industrial production growth has a positive relation with the time-varying parameter on the market, investment, and momentum factors, and a negative relation with the size, value, and profit factors, with the effect on the volatility factor split equally. Thus, when output growth is positive, firms take on greater systematic risk and increase investment, while the momentum effect is enhanced. When output growth is negative, the size and value effects lean towards firms with large growth. The VIX has a negative relation with the time-varying parameters on the market, size, investment, volatility and momentum factors, while for the remaining factors, the sign of the coefficient is split equally between positive and negative. This implies that falling market risk coincides with greater systematic risk and a move towards high volatility, low investment and value firms, again, with enhanced momentum effects. Thus, these coefficient signs appear to suggest a limited connection between economic and market risk and the asset pricing risk factors.

Table 6 presents the corresponding results for small firms. We would expect the same general pattern for small stocks—in terms of the coefficient signs—as for large stocks, although, as small stocks are regarded as riskier, we would expect the magnitude of the coefficient to be larger. However, here, we can see a different picture emerging in the interrelation between the time-varying coefficients and industrial production growth and VIX. Industrial production growth has a negative relation across all the factors, while the VIX has a positive relation with the value, investment, and momentum factors, and a negative relation with the size, profit, and volatility factors. This suggests that for industrial production, there is a greater correspondence between the sign of the relation and our expectations, with only the profit factor running counter to those expectations. An increase in economic risk is associated with small value and less profitable stocks. For the VIX, the nature of the results is similar, although less strong, with three of the coefficients indicating the expected sign between the time-varying factor loading and VIX for value, profit, and investment. Thus, the behaviour of small stocks appears to be more closely linked to risk, in terms of the coefficient sign, when compared to large stocks.

Across these two tables, the evidence linking a change in macroeconomic or market risk with the time-varying behaviour of the factors, and whether their impact on stock returns changes, is limited. Specifically, the nature of the signs between economic state risk variables and portfolio risk factors is not fully consistent across the full range of portfolios and factors considered, although there is greater support for small stocks. Nonetheless, these results must cast some doubt on the usefulness of the factors in capturing the nature of risk and their use in an asset pricing model.

	Value Portfolio—High							**Value Portfolio—Low**						
	Mkt	**SMB**	**HML**	**Prof**	**Inv**	**Vol**	**Mom**	**Mkt**	**SMB**	**HML**	**Prof**	**Inv**	**Vol**	**Mom**
IP	0.951	0.318	−0.752	−0.021	1.168	−0.357	−0.051	0.113	−0.156	−0.234	−0.308	1.439	0.046	0.070
	(5.33)	(1.23)	(−2.81)	(−0.06)	(3.49)	(−5.23)	(−0.74)	(2.11)	(−2.55)	(−2.90)	(−2.15)	(10.48)	(1.91)	(1.41)
VIX	0.212	0.503	0.184	0.163	0.707	−0.206	−0.086	−0.200	−0.206	−0.106	0.009	−0.018	−0.035	0.061
	(2.16)	(3.55)	(1.24)	(0.84)	(3.84)	(−5.48)	(−2.25)	(−6.77)	(−6.11)	(−2.38)	(0.12)	(−0.23)	(−2.65)	(2.22)
	Profit Portfolio—High							Profit Portfolio—Low						
IP	0.367	−0.464	−0.219	−0.064	0.639	0.087	−0.139	−0.207	−0.252	0.734	−1.106	−0.670	−0.156	0.394
	(6.81)	(−6.73)	(−3.26)	(−0.68)	(6.63)	(3.75)	(−2.88)	(−2.25)	(−2.57)	(6.72)	(−8.17)	(−3.77)	(−5.46)	(4.75)
VIX	−0.002	−0.216	−0.090	0.230	0.147	−0.012	0.102	−0.136	−0.313	0.110	−0.133	−0.003	0.075	−0.064
	(−0.06)	(−5.68)	(−2.42)	(4.42)	(2.77)	(−0.95)	(3.80)	(−2.69)	(−5.80)	(1.84)	(−1.78)	(−0.03)	(4.78)	(−1.39)
	Investment Portfolio—High							Investment Portfolio—Low						
IP	0.093	0.056	0.335	−0.954	0.992	−0.006	0.152	0.067	−0.014	0.566	−1.240	−0.017	0.079	−0.178
	(1.30)	(0.56)	(2.26)	(−5.45)	(10.45)	(−0.16)	(2.70)	(0.78)	(−0.24)	(3.63)	(−8.63)	(−0.13)	(3.19)	(−3.03)
VIX	−0.344	−0.181	0.132	−0.524	−0.342	−0.44	−0.064	0.149	0.030	0.220	−0.618	0.249	−0.018	−0.128
	(−8.74)	(−3.31)	(1.62)	(−5.44)	(−6.54)	(−2.08)	(−2.05)	(3.14)	(0.94)	(2.56)	(−7.81)	(3.50)	(−1.28)	(−3.95)
	Momentum Portfolio—High							Momentum Portfolio—Low						
IP	0.218	−0.933	−0.481	0.339	0.680	−0.030	0.098	−0.344	0.805	−0.598	0.083	0.135	−0.025	−0.650
	(5.08)	(−6.88)	(−3.22)	(2.49)	(5.37)	(−5.16)	(0.82)	(−4.98)	(6.57)	(−3.19)	(0.23)	(0.51)	(−2.91)	(−4.03)
VIX	−0.152	−0.557	−0.078	−0.347	0.059	0.002	−0.229	−0.239	0.113	−0.368	0.503	−0.380	0.001	−0.481
	(−6.45)	(−7.46)	(−0.95)	(−4.62)	(0.85)	(0.66)	(−3.51)	(−6.28)	(1.67)	(−3.56)	(2.49)	(−2.60)	(1.41)	(−5.41)
	Quality Portfolio—High							Quality Portfolio—Low						
IP	−0.169	−0.709	−0.584	−0.002	0.295	0.002	0.012	−0.283	0.507	−0.207	0.790	0.249	0.245	0.095
	(−3.58)	(−6.41)	(−4.55)	(−0.01)	(5.30)	(0.44)	(0.23)	(−5.52)	(4.96)	(−1.62)	(4.09)	(2.02)	(3.23)	(1.02)
VIX	−0.239	−0.108	−0.282	0.001	−0.032	0.001	0.001	−0.210	0.001	0.129	−0.024	−0.349	−0.001	−0.129
	(−9.19)	(−1.77)	(−3.99)	(0.03)	(−1.04)	(1.98)	(3.97)	(−7.42)	(0.11)	(1.84)	(−0.23)	(−5.14)	(−0.54)	(−2.54)

Notes: Entries are coefficient values (and Newey–West t-statistics) of regressing the time-varying factor coefficients on the growth rate of industrial production (IP) and implied volatility (VIX). The VIX coefficients are multiplied by 100.

Table 5 Are Factor Coefficients Related to Economic and Market Risk?—Large Firms.

	HML—HIGH							**HML—LOW**						
	Mkt	**SMB**	**HML**	**Prof**	**Inv**	**Vol**	**Mom**	**Mkt**	**SMB**	**HML**	**Prof**	**Inv**	**Vol**	**Mom**
IP	0.284	−0.796	−0.031	−1.072	0.558	0.181	−0.391	−0.037	−0.118	0.103	−2.088	−0.959	0.005	−0.586
	(2.63)	(−7.10)	(−0.12)	(−6.31)	(4.36)	(4.62)	(−4.34)	(−0.40)	(−1.20)	(0.33)	(−9.76)	(−4.36)	(0.13)	(−6.29)
VIX	−0.007	−0.187	0.221	−0.166	−0.044	0.023	0.006	0.280	0.081	0.188	−0.361	0.203	−0.121	0.070
	(−0.12)	(−3.03)	(1.50)	(−1.77)	(−0.63)	(1.08)	(0.12)	(5.45)	(1.50)	(1.11)	(−3.07)	(1.68)	(−5.44)	(1.37)
	PROFIT—HIGH							PROFIT—LOW						
IP	−0.338	−0.288	0.209	−1.620	−1.619	−0.103	−0.333	0.181	−0.499	−0.749	−0.584	−0.316	0.141	−0.866
	(−4.44)	(−2.73)	(0.72)	(−8.11)	(−11.03)	(−2.50)	(−4.75)	(2.13)	(−6.62)	(−2.53)	(−2.99)	(−1.94)	(3.05)	(−7.59)
VIX	0.032	−0.128	0.205	−0.425	0.070	−0.026	−0.033	0.164	−0.031	0.005	−0.064	0.214	−0.114	0.133
	(0.66)	(−2.21)	(1.28)	(−3.86)	(0.87)	(−1.15)	(−0.85)	(3.51)	(−0.75)	(0.03)	(−0.60)	(2.40)	(−4.47)	(2.12)
	INVESTMENT—HIGH							INVESTMENT—LOW						
IP	−0.136	−0.487	−0.130	−1.306	−1.262	0.044	−0.710	−0.108	−0.419	−0.361	−1.018	−0.255	−0.042	−0.385
	(−1.55)	(−4.81)	(−0.39)	(−7.21)	(−7.99)	(1.01)	(−7.96)	(−1.62)	(−5.96)	(−1.68)	(−5.64)	(−1.14)	(−1.03)	(−3.66)
VIX	0.3349	−0.045	0.099	−0.191	0.291	−0.085	0.098	−0.145	−0.259	0.012	−0.094	−0.303	−0.111	0.016
	(7.23)	(−0.82)	(0.54)	(−1.92)	(3.34)	(−3.55)	(1.98)	(−3.92)	(−6.68)	(0.10)	(−0.95)	(−2.46)	(−4.93)	(2.79)
	MOM—HIGH							MOM—LOW						
IP	−0.083	0.644	−0.071	−0.988	−0.522	−0.026	−0.403	−0.138	−0.881	−0.380	−0.641	−0.150	−0.013	−0.312
	(−1.74)	(4.33)	(−0.67)	(−5.87)	(−5.28)	(−2.80)	(−6.12)	(−2.23)	(−9.84)	(−1.45)	(−2.62)	(−1.12)	(−1.79)	(−4.24)
VIX	−0.036	0.175	0.175	−0.099	−0.136	−0.018	−0.039	−0.005	−0.492	−0.185	−0.548	0.340	0.001	0.151
	(−1.40)	(2.13)	(3.01)	(−1.07)	(−2.49)	(−3.53)	(−1.06)	(−0.17)	(−9.99)	(−1.28)	(−4.06)	(4.59)	(0.02)	(3.72)
	QMJ—HIGH							QMJ—LOW						
IP	−0.001	−0.209	−0.065	−0.895	−0.734	−0.001	−0.163	−0.031	0.386	−1.215	1.900	−0.747	−0.012	−0.273
	(−0.01)	(−2.79)	(−0.34)	(−5.59)	(−6.04)	(−0.12)	(−4.35)	(−0.43)	(3.41)	(−5.58)	(9.16)	(−5.75)	(−1.34)	(−3.53)
VIX	0.142	−0.021	0.182	−0.394	0.085	−0.001	−0.063	−0.062	0.038	−0.905	0.589	0.329	0.001	0.263
	(5.95)	(−0.51)	(1.72)	(−4.47)	(1.27)	(−2.21)	(−3.07)	(−1.54)	(−0.60)	(−7.54)	(5.16)	(4.60)	(1.73)	(6.18)

Notes: Entries are coefficient values (and Newey–West t-statistics) of regressing the time-varying factor coefficients on the growth rate of industrial production (IP) and implied volatility (VIX). The VIX coefficients are multiplied by 100.

Table 6 Are Factor Coefficients Related to Economic and Market Risk?—Small Firms.

5. Trading Rule

As a final exercise, we wish to examine whether we can use the information captured in the expected returns models to develop a trading rule that allows us to switch between portfolios. To this end, we construct the one-step-ahead expected return based on the estimated rolling coefficients and the value of each factor. Thus, the expected return is given as

$$r_{t+1} = \Sigma_{i=1}{}^{k} \hat{\beta}_i x_{it}, \tag{2}$$

where r_{t+1} refers to next period's portfolio return, $\hat{\beta}_i$ is the fitted coefficient, and x_{it} are the stock market factors. Hence, we are only using past information to explain the future value of the portfolio return, thus producing an out-of-sample forecast for returns.

Having generated the expected returns for each portfolio, we then develop a trading rule for each style of investment type. Taking the value portfolios as an example, we compare the estimated expected return for the portfolios constructed across large and small firms and high and low book-to-market firms. These portfolios can be viewed as passive style instruments where an investor holds stock according to these characteristics. The trading rule then takes the form of whichever of the four different portfolios (large/high, large/low, small/high, and small/low) has the largest expected return as obtained from Equation (2) for a given month; we then hold that portfolio. We reassess the highest expected return each month over the period for which we have estimated coefficients (1995 month 1 to 2016 month 10). Thus, we have a series that contains the active portfolio return. From the passive and active portfolios, we can examine the average return and the Sharpe ratio to determine whether the asset pricing models contain useful information.

We conduct this active trading rule for expected returns generated by the CAPM, FF3, FF5, and F7 models. Thus, in addition to considering whether we can add factors to the FF5 model, we also raise the issue of whether the extra factors in the FF5 model are needed over the FF3 model. We ask this given the limited significance of the profit and investment factors noted in Tables 1 and 2. In deciding whether this active approach—and, thus, the asset pricing models that generate it—has any value, we compare the obtained trading returns and the Sharpe ratio to those obtained by just holding any of the four passive portfolios. We conduct the trading rules for the portfolios that are double-sorted by size and value, investment, profit, momentum, and quality.[8]

The forecast results are presented in Table 7 and demonstrate an interesting pattern across the different portfolios sets. Taking an overall view, we can see that the small/high portfolio achieves the highest return and Sharpe ratio for the book-to-market and quality portfolios, the small/low portfolio achieves the highest return, while the big/low portfolio achieves the highest Sharpe ratio for the investment portfolios. For the profit portfolios, the active FF5 model achieves the highest return but the passive big/high achieves the highest Sharpe ratio. For the momentum portfolio, the FF3 model achieves the highest return while the big/high achieves the highest Sharpe ratio. Thus, purely on the best-performing trading measures, the passive portfolios, as a group, outperform the active portfolios. However, across the different trading exercises, the same passive portfolio does not perform equally well. For example, the small/high portfolio that performs the best for two portfolios performs the worst for the investment portfolios. Equally, the big/low and small/low style types also perform the worst for a set of portfolios. Thus, there is no simple rule to follow in that respect.

However, while the active portfolios may not be preferred in terms of achieving the highest trading return or Sharpe ratio for each investment type (except on one occasion for both the FF3 and FF5), they do perform consistently well. Notably, the FF3 model is ranked second five times, while the FF5 model is ranked second once and third five times (both these models also rank fourth once), in addition to each being first once. The F7 model is ranked third on two occasions and fourth once. Moreover, the expected return models never perform the worst (although the CAPM does once) and their relative performance is similar, especially for the FF3 and FF5 models across all portfolios. Thus, the active portfolio approaches perform consistently better than the passive

[8] Results for the other portfolios are available upon request but provide qualitatively similar results and are thus not reported for space considerations.

style approaches. Moreover, between the four active trading approaches, the FF3 and FF5 models perform best while the CAPM performs worst. These results thus suggest that a multifactor model does enhance model accuracy, with the FF3, FF5, and F7 outperforming the CAPM. Equally, the FF3 and FF5 models outperform the F7 model. These results do not support the need for the inclusion of momentum and volatility as additional factors. Moreover, it is not clear from these results that the profit and investment factors in the FF5 model contribute much to our understanding of asset price movements or benefit investors beyond the results found with the FF3 model.

	CAPM	**FF3**	**FF5**	**F7**	**BH**	**BL**	**SH**	**SL**
				Book-to-Market Portfolios				
Ret	0.910	1.216 †	1.138 ‡	1.062	0.915	0.918	1.235 *	0.820 ⸸
SR	0.145 ⸸	0.210 †	0.191	0.200	0.172	0.208 ‡	0.220 *	0.177
				Investment Portfolios				
Ret	0.893	1.112 †	1.022 ‡	1.012	0.859	0.977	0.812 ⸸	1.230 *
SR	0.155	0.196 †	0.184	0.182	0.163	0.231 *	0.124 ⸸	0.189 ‡
				Profit Portfolios				
Ret	1.097	1.129	1.250 *	1.155 ‡	0.998	0.664 ⸸	1.239 †	0.903
SR	0.181	0.207	0.232 †	0.214	0.246 *	0.114 ⸸	0.229 ‡	0.127
				Momentum Portfolios				
Ret	0.928	1.365 *	1.260	1.262 ‡	1.006	0.659 ⸸	1.326 †	0.798
SR	0.144	0.195	0.200 ‡	0.188	0.220 *	0.100	0.214 †	0.099 ⸸
				Quality Portfolios				
Ret	0.794	0.914 †	0.910 ‡	0.881	0.740	0.428 ⸸	1.073 *	0.505
SR	0.118	0.149	0.151 ‡	0.150	0.180 †	0.073	0.213 *	0.064 ⸸

Notes: Entries refer to the return (Ret) and Sharpe ratio (SR) of the different trading strategies either active based on the asset pricing models (CAPM, Fama–French 3- and 5-factor model and 7-factor model) or passive (holding big/high, big/low, small/high, and small low) for the different portfolios. The notation indicates ranking as * best, † second, ‡ third, and ⸸ worst.

Table 7 Trading Rule Performance.

6. Summary and Conclusions

This paper has sought to examine whether adding momentum and volatility factors to the FF5 asset pricing model improves our understanding of asset pricing. Crucially, we are not interested in whether this provides a better statistical fit to the stock return data but whether these variables have additional economic content. Specifically, we are interested in whether these factors explain portfolios beyond those constructed from the factor itself, whether the nature of the coefficient make economic sense, and whether their inclusion enhances a trading rule approach.

We begin by examining the ability of the FF5 model to capture the behaviour of a wide range of portfolios. Our results show that across the range of portfolios, the intercept (alpha) term remains statistically significant for more than half of the portfolios while the factors themselves are not significant throughout. We use the FF5 model as the baseline model as it has effectively become the state-of-the-art model, supplanting the FF3 (Fama–French three-factor) model. However, our results suggest that this model may lack a complete explanation of stock return behaviour. Moreover, the research suggests a range of alternative factors, with the implication that further factors could be added to the model. That said, the danger of this approach is including factors to account for every suggested anomaly and leading to what is known as the factor zoo. Therefore, we consider two of the more prominent factors examined within the literature: momentum and low volatility. Our view is that for these factors to merit inclusion in the asset pricing model, they must explain stock return behaviour beyond portfolios based on the factor, have meaningful coefficient signs, and improve out-of-sample trading.

In terms of these three criteria, our results suggest the following. First, the momentum and volatility factors exhibit little statistical significance across the full range of portfolios. For large firms, the volatility factor is only significant for one portfolio, while for small firms, only for four portfolios. For the momentum factor, excluding the momentum portfolios, there is slightly greater evidence of statistical significance, being significant for two large-firm portfolios and eight small-firm portfolios. However, even for the momentum portfolio, the estimated coefficient is similar across the high and low portfolios for a given characteristic in several instances. This suggests little economic content in the factor. Second, we further consider the nature of the coefficients by introducing time variation through a rolling regression. Here, the results reveal not only evidence of time variation within the coefficient values but also that the sign of the coefficients changes over the sample period. This raises issues concerning the economic interpretation of the factors. Specifically, where these empirical factors proxy for unknown economic risk factors, they should exhibit consistent behaviour on expected returns. However, this result not only applies to the momentum and volatility factors but also across the full range of factors in the FF5 model, where the coefficient changes the economic meaning of the factor. Third, examining the trading rule and comparing the FF5 and F7 models, the former model provides the most consistent performance, although is rarely the preferred model, while the latter model performs to a similar but slightly lower level. In implementing this trading rule, we also consider the CAPM and FF3 models. The CAPM produces a relatively weak trading performance, however, the FF3 performs similarly to the FF5 model and indeed could be argued to be superior.

Our results suggest that the FF5 (and FF3) asset pricing model adds value in explaining stock return behaviour over the CAPM, but adding momentum and volatility does not. Moreover, our results question whether the FF5 provides any information over and above the FF3 model as both models achieve a similar trading performance. However, there remains open issues. Perhaps the biggest of these issues is the behaviour of the factor coefficients over time. Notably, time variation in these factors changes their economic interpretation, for example, where the market factor switches from above to below the value of 1. Understanding why these changes occur is of key importance—a model using lagged economic growth and VIX does not appear to provide such an explanation. Nonetheless, it is hoped the results presented here are useful in advancing the asset pricing debate and whether momentum and volatility factors play a role, as well as posing questions for future research.

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